

"What you do today can improve all your tomorrows"-Ralph Marston

HW: "Difference Quotient" worksheet #1-4

Aim: What is the Difference Quotient?

Warm Up: Which of the following are continuous at $x = 1$?

yes (a) $f(x) = \begin{cases} 5 & x = 1 \\ 2x + 3 & x \neq 1 \end{cases}$ $\xrightarrow{x=1} 5$ $2(1)+3 \rightarrow 5$ yes

yes (b) $f(x) = \begin{cases} 4x & x = 1 \\ \frac{2x^2-2}{x-1} & x \neq 1 \end{cases}$ $4(1) \rightarrow 4$ $\rightarrow 4$

yes (c) $f(x) = \begin{cases} 0 & x = 1 \\ \frac{x-1}{\sqrt{1-x}} & x \neq 1 \end{cases}$ $\rightarrow 0$ $\rightarrow 0$

b) $\frac{2x^2-2}{x-1} = \frac{2(x^2-1)}{x-1} = \frac{2(x+1)\cancel{(x-1)}}{\cancel{x-1}} = 2(x+1) = 2(1+1) = 4$

c) $\frac{x-1}{\sqrt{1-x}} \cdot \frac{\sqrt{1-x}}{\sqrt{1-x}} = \frac{\cancel{(x-1)}(\sqrt{1-x})}{\cancel{1-x}} = -\sqrt{1-x} = -\sqrt{1-1} = 0$

1. Given $f(x) = 4x^2$, find the following and simplify.

(a). $f(x+h)$

$$4(x+h)^2$$

$$4(x^2 + 2xh + h^2)$$

$$\boxed{4x^2 + 8xh + 4h^2}$$

(b). $f(x+h) - f(x)$

$$\underline{4x^2 + 8xh + 4h^2} - \underline{4x^2}$$

$$\boxed{8xh + 4h^2}$$

(c). $\frac{f(x+h) - f(x)}{h}$

$$\frac{8xh + 4h^2}{h} = \frac{\cancel{h}(8x + 4h)}{\cancel{h}}$$

$$= \boxed{8x + 4h}$$

Find $\frac{f(x+h)-f(x)}{h}$ for each of the following.

1. $f(x) = x^2 - 5x - 1$

$$\frac{(x+h)^2 - 5(x+h) - 1 - (x^2 - 5x - 1)}{h}$$

$$\frac{\cancel{x^2} + 2xh + \cancel{h^2} - \cancel{5x} - 5h - \cancel{1} - \cancel{x^2} + \cancel{5x} + \cancel{1}}{h}$$

$$\frac{2xh + h^2 - 5h}{h} = \boxed{2x + h - 5}$$

2. $f(x) = 3x^2 - 4x$

3. $f(x) = x^3 - 4x^2 + 5x$

$$\frac{f(x+h)-f(x)}{h}$$

4. $f(x) = \frac{1}{x}$

$$\frac{\text{LCD: } x(x+h)}{x(x+h)} \quad \frac{\cancel{x(x+h)} \cdot \frac{1}{\cancel{x+h}} - \frac{1}{\cancel{x}} \cdot \cancel{x(x+h)}}{\frac{h}{1} x(x+h)} = \frac{x - (x+h)}{hx(x+h)} = \frac{x - x - h}{hx(x+h)}$$

$$= \frac{-h}{hx(x+h)} = \boxed{\frac{-1}{x(x+h)}}$$

$$3) \quad f(x) = \sqrt{7x-8}$$

$$\frac{f(x+h) - f(x)}{h} = \frac{\sqrt{7(x+h)-8} - \sqrt{7x-8}}{h} \cdot \frac{\sqrt{7x+7h-8} + \sqrt{7x-8}}{\sqrt{7x+7h-8} + \sqrt{7x-8}}$$

$$\frac{7x+7h-8 - (7x-8)}{h(\sqrt{7x+7h-8} + \sqrt{7x-8})} = \frac{7h}{h(\sqrt{7x+7h-8} + \sqrt{7x-8})}$$

$$= \frac{7}{\sqrt{7x+7h-8} + \sqrt{7x-8}}$$

$$(x+h)^2 = x^2 + 2xh + h^2$$

$$\begin{aligned}(x+h)^3 &= (x+h)(x^2 + 2xh + h^2) \\&= x^3 + 2x^2h + xh^2 + x^2h + 2xh^2 + h^3 \\&= x^3 + 3x^2h + 3xh^2 + h^3\end{aligned}$$